GENERAL ENGINEERING LABORATORY

TEMPERATURE DISTRIBUTION IN VLF BUSHING

By

Mrs. F. F. Buckland
General Engineering Laboratory

June 1, 1962

GENERAL ELECTRIC

The information contained in this report has been prepared for use by the General Electric Company and its employees. No distribution should be made outside the Company except when indicated below:

TEMPERATURE DISTRIBUTION IN VLF BUSHING

Mrs. F. F. Buckland

June 1, 1962

TABLE OF CONTENTS

I. ABSTRACT

II. CONCLUSIONS

III. SYMBOLS

IV. INTRODUCTION

V. THE METHOD

VI. SAMPLE CALCULATION FOR COPPER BUSHING

VII. RESULTS OF CALCULATION

I. ABSTRACT:

Temperatures are calculated for original bushing design, and modifications to lower the temperatures are proposed. The basis of the recommendations is the experience on the Cutler bushings and a Pittsfield test with a high current. The calculation method is described, and references to sources are given.

II. CONCLUSIONS:

Putting 2540 amperes through a bushing of the original design causes the current-carrying tube to run much too hot, with subsequent damage to the nitrile rubber O-rings, and loss of the SF_6. It is possible to so treat the surface of copper or aluminum that the emissivity is increased substantially, with consequent good heat transfer to the porcelain and reduction in tube temperature. There is a chance that the difficulty of shipping the tube will mar the postulated surface, and the temperature will rise again beyond acceptable limits.

III. SYMBOLS

A_δ = area in which skin effect is generated, sq. in.

A_s = surface area from which heat is dissipated, sq. in.

D = duct diameter, in inches for heat generation, in feet for air cooling.

f_δ = frequency

f_p = coefficient of friction

h_c = heat-transfer coefficient for convection, watts/(sq. in.) deg. C.

h_r = heat-transfer coefficient for radiation, watts/(sq. in.) deg. C.

I = current, amperes

L = length of heat flow path, inches

L_p = length of air flow path, feet

p = pressure drop, inches of water

q = heat dissipated, watts

t = temperature, deg. C

V = velocity, fph or fps

Greek

δ = depth of skin effect, inches

Δ = difference (in temperature) deg. C

ϵ = emissivity of surface, dimensionless

ρ_e = electrical resistivity, ohm-cm.

ρ = density, lb./(cu. ft.)

Subscripts

1, 2, 3, 4 apply to area under consideration.

IV. INTRODUCTION

The problem of calculating the temperatures of components of the VLF bushing proposed for the South Pacific presents several challenging features. The first problem is what happens when the current is raised from 750 amperes to 2540 amperes. Obviously the temperatures will be higher, and in the high ambient now specified, 120 F (≡ 49 C) what will be the effect on the design? There seem to be two critical factors, one the effect of higher temperatures on the sealing effectiveness of the O-rings, and the other the effect of differential expansion.

In using an electrical-conductor grade aluminum for heat generation reasons, instead of the present structural aluminum, the allowable stresses will have to be reduced. Present plans are to reduce the clamping force.

It is now stated (as of May 28, 1962) that the Helix House will be pressurized at 1/2 inch of water so there will be a head available to drive cooling air through the tube.

V. THE METHOD

The enclosed reprint of an AIEE paper of mine, "Basic Heat-Transfer Data in Electron Tube Operation" shows a simple thermal circuit. It also gives curves for radiation, free convection, and forced convection, as well as the equation for pressure drop.

Because of the mixture of heat transfer, fluid flow, and electricity in this problem, I have used subscripts on some of the standard symbols. The meaning of "f" in electricity is frequency, and in fluid flow is friction factor. The meaning of "ρ" in electricity is resistivity, and in heat transfer is density.

The sequence of operations is as follows:

1. Calculate the heat to be dissipated, from the relation

$$q = I^2 R = \frac{I^2 \rho L}{\pi \delta D \times 2.54} \quad \text{, watts}$$

$$\text{where } \delta = \frac{1}{2.54 \times 2\pi} \left(\frac{\rho_\delta \times 10^9}{f} \right)^{1/2} \quad \text{, inches}$$

and the electrical resistivity for pure aluminum is

$$\rho_{al} = 2.828 \times 10^{-6} \left[1 + 0.0039 \, (t - 20^\circ \text{ C}) \right] \quad \text{, ohm-cm}$$

For pure copper, it is

$$\rho_{cu} = 1.721 \times 10^{-6} \left[1 + 0.00393 \, (t - 20^\circ \text{ C}) \right] \quad \text{, ohm-cm.}$$

It is necessary to estimate the temperature rise, and to recalculate if it is seriously in error.

2. Heat dissipation

The watts calculated in (1) are dissipated by radiation and convection from the several surfaces, according to the relation

$$\frac{q}{A_s} = (h_r + h_c) \, \Delta t$$

The radiation coefficient h_r is plotted in Figure 2 of my paper, for an emissivity of 0.9.

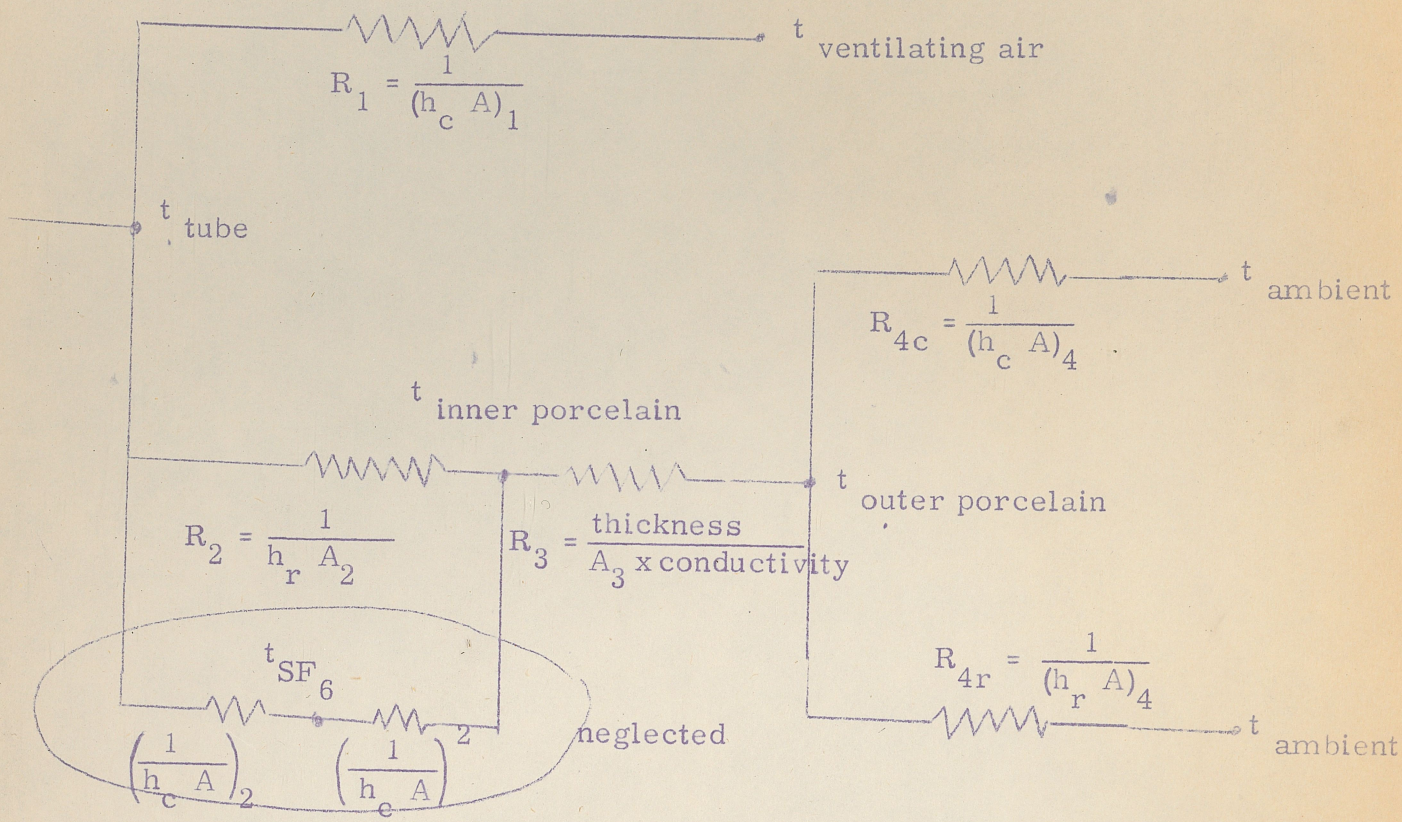
Emissivity of the tube is the property that is critical in this application. The higher the emissivity, the better the ability to radiate. Slightly oxidized aluminum has an emissivity of 0.10, and strongly anodized 0.87 (Ref. 1). Tarnished copper has an emissivity of 0.5 (Ref. 2). Thus, radiation is 8.7 times as effective with anodized aluminum, and 1.8 times with copper.

The free-convection coefficient h_c for air is plotted in Figure 3 of my paper. This coefficient is used for the outer surface of the porcelain only, since forced convection is used inside the tube. No account has been taken of free convection effects in the SF_6 area, because of the difficulty in interpreting the flow pattern. Due to the interruption at the flange, the circulation will be in two zones, with the gas flowing up along the tube and down along the porcelain. As radiation is the dominant mode of heat transfer, the contribution by the SF_6 may be neglected. If a detailed analysis is needed, it may be made by consulting McAdams' "Heat Transmission," 3rd edition, p. 181, "Vertical, Enclosed Air Spaces." The equations contain the thermal properties, which may be obtained from the manufacturer.

Reference 1. Trans. Am. Soc. Htg. and Ventilating Eng., Vol. 45, 1939, pp. 179-194.

2. Temperature, Its Measurement and Control, p. 1182.

To use the foregoing relation, it is of advantage to construct a thermal circuit. This circuit can be vastly complicated to take into account everything that is happening in all parts of the bushing, but a simplified circuit that gives the principal temperatures is as follows:



This circuit is for radial flow only, but axial flow may be added by changing the temperature of the internal cooling air and using the same circuit.

The part of the circuit that extends from the inner porcelain to the ambient air may be replaced with very little error by a drop of 10°C. Both my calculations and the results inferred from the Pittsfield tests bear out this conclusion.

VI. SAMPLE CALCULATION FOR COPPER BUSHING

I = 2540 amperes

f_δ = 15000 cycles/sec.,

assumed copper temperature = 104 C

bushing OD 36 in.

Tube ID 9.75 in., OD 10.25 in.

Av. D of inner porcelain = 22 in.

Av. thickness of porcelain = 1.25 in.

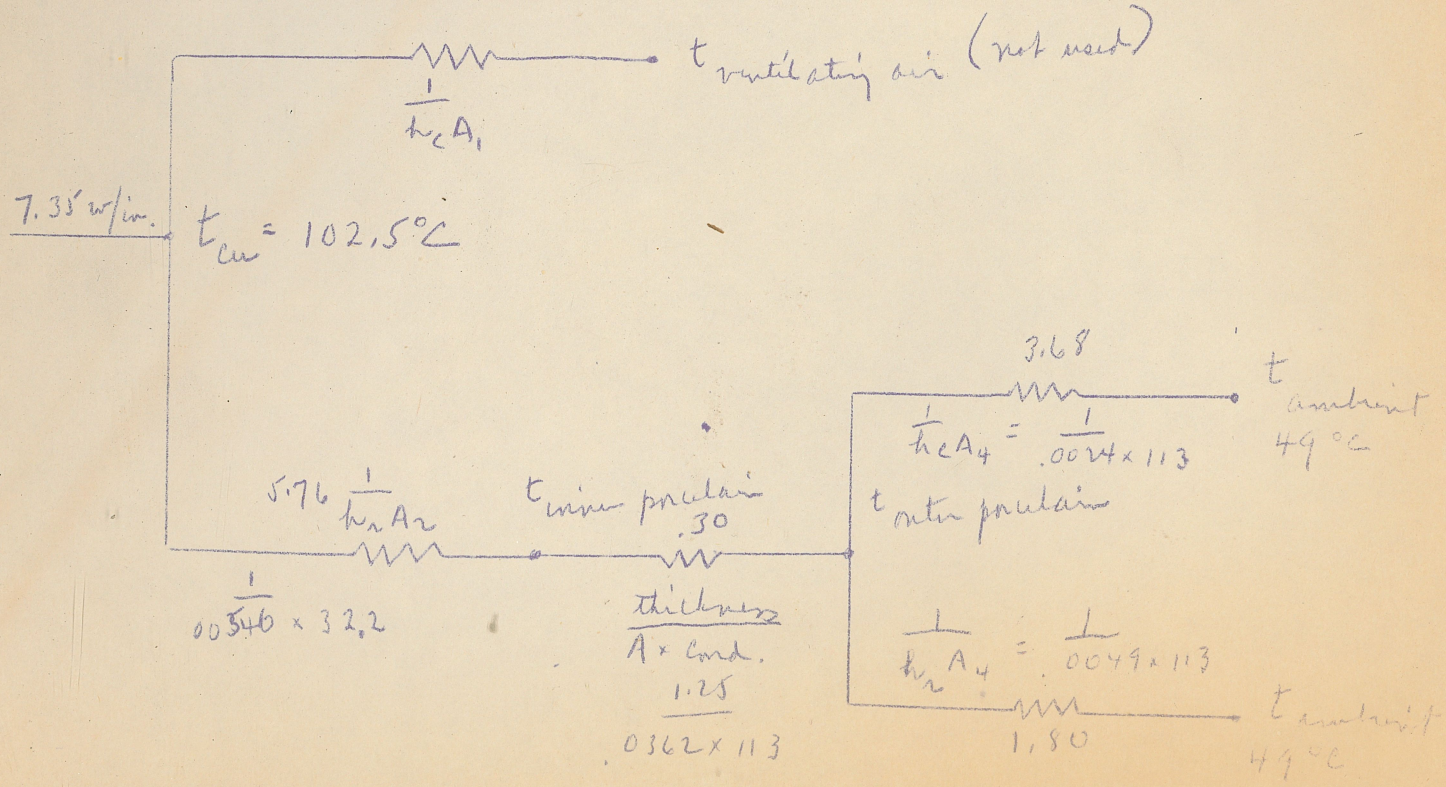
Conductivity of porcelain = .0362 w/(sq. in) deg. C/in.

$$\delta = \frac{1}{5.08\pi}\left(\frac{\rho \times 10^9}{f_\delta}\right)^{1/2}$$

$$\rho_{cu} = 1.721 \times 10^{-6}\left[1 + .00393\ (104 - 20)\right] = 1.721 \times 1.330 = 2.29 \times 10^{-6} \text{ ohm-cm}$$

$$\delta = \frac{1}{5.08\pi}\left(\frac{2.29 \times 10^3}{15000}\right)^{1/2} = \frac{1}{5.08\pi}(.1527)^{1/2} = .0245 \text{ in.}$$

$$q = I^2 R = \frac{I^2 \rho L}{2.54\ \pi\ \delta\ D} = \frac{2.54^2 \times 10^6 \times 2.29 \times 10^{-6} \times 1}{2.54\ \pi \times .0245 \times 10.25} = \frac{14.8}{2.01} = 7.37 \text{ watts/in.}$$



VII. RESULTS OF CALCULATION

The sample calculation of Section VI indicates a copper temperature of 102.5 C, without forced air cooling. The emissivity used for copper, inside porcelain, and outer porcelain was 0.9. In order to reach the high emissivity for the copper, it must be black oxidized. Such surface treatment will be unacceptable in the region of electrical connections, because of the high electrical resistance.

Forced convection will drop the temperature to 70 C. To calculate this part of the thermal circuits, Figure 5 and the pressure drop equation of my paper are used. A flow of 396 cfm will result in a pressure drop in the tube of 10% of the 1/2 inch of water assumed. Inlet and exit losses will be substantial, but the flow may still need to be throttled in order not to waste air.

Using half the bushing outside diameter for the tapered parts of the bushing results in 1.5 C additional rise.

Recalculating for a frequency of 30,000 cycles results in 28 C additional rise.

AIEE PROCEEDINGS SECTION T 1-227

A Preprint from AIEE TRANSACTIONS, Volume 70, 1951. All rights reserved. Publications Office, 500 5th Avenue, New York 18, N. Y. Printed in the United States of America.

Published by the American Institute of Electrical Engineers. H. H. Henline, Secretary; K. B. McEachron, Chairman, Publication Committee; Charles S. Rich, Editor

Basic Heat-Transfer Data in Electron Tube Operation

MRS. B. O. BUCKLAND
MEMBER AIEE

SUCCESSFUL operation of an electron tube depends on its heat dissipating characteristics as well as on its electronic characteristics. The tube energy is not all useful, unfortunately, and the part of the total energy that appears as heat must be transferred to the environment. Heat is generated at several sources; the primary sources are at the hottest temperatures, while various structural components assume temperatures intermediate between source temperatures and ambient. All the temperatures are of interest to the tube designer, because of his knowledge of the effect of temperature on the materials with which he works as well as on electron emission.

This paper is primarily concerned with operation of the tube in a known environment, although the topics covered also are of importance in tube design. Evaluation of thermal resistance is the principal topic. The equations and formulas to be presented are available in the literature, which will be cited, but not always in a form convenient for use in electrical problems.

It is possible to estimate the temperature at any location in a tube due to the losses it generates, by means of the well-known device of a thermal circuit. James Clerk Maxwell[1] was the first to point out the analogy between conduction of electricity and conduction of heat. He wrote, in part,

"The analogy between the theory of the conduction of electricity and that of the conduction of heat is at first sight almost complete. If we take two systems geometrically similar, and such that the conductivity for heat at any part of the first is proportional to the conductivity for electricity at the corresponding part of the second, and if we also make the temperature at any part of the first proportional to the electrical potential at the corresponding point of the second, then the flow of heat across any area of the first will be proportional to the flow of electricity across the corresponding area of the second .... Flow of electricity corresponds to flow of heat, the electric potential to temperature, and electricity tends to flow from places of high to places of low potential, exactly as heat tends to flow from places of high to places of low temperature."

The thermal circuit is thus analogous to an electric circuit, with current replaced by heat flow rate, voltage replaced by temperature difference, and resistance replaced by thermal resistance. It is no longer considered necessary to have geometrical similarity, and the procedure may be extended to the case of transient heat flow, by replacing electric capacity with thermal capacity.

The natural mode of heat dissipation from the surface is by free convection and radiation; at room temperature the magnitude of these two effects is about the same. When the tube runs too hot, recourse must be had to forced convection, in which a cooling fluid, usually air or water, is forced over the surface by mechanical means. In some special applications the tube is required to operate in a fixed temperature range, so that sometimes it is cooled and sometimes it is heated.

Thermal Circuit for Steady State

The complete thermal circuit for heat dissipation from a tube consists of a network which is evaluated in the ordinary manner by means of Kirchhoff's laws or similar theorems. However, any element may be evaluated separately if heat flows or temperatures up to the point of consideration are known.

The first circuit to be illustrated gives the temperature of a tube envelope, cooled by radiation and free convection, when the heat generation and the ambient temperature are known, see Figure 1.

The equation of this circuit gives the temperature rise of the surface above ambient air as:

$$t_s - t_a = \frac{q}{A(h_c + h_r)} \qquad (1)$$

which is directly analogous to $E = IR$.

When temperatures t are expressed in degrees centigrade, heat flow q in watts, and area A in square inches, the units of the surface heat-transfer coefficients, h_c for convection and h_r for radiation, are watts, square inches, degrees centigrade. It is to be noted that the coefficients are of the nature of conductances, so that where resistances are more convenient, the reciprocals of the heat transfer coefficients are to be used.

The heat generated in the tube must be known; the losses are the space charge (anode) loss, the filament power and the grid loss. Any other expected losses should be included.

To solve the circuit of Figure 1, a certain amount of cut-and-try is required, because the heat-transfer coefficients are functions of the temperature. This situation is analogous to solving an electric circuit in which the resistances are functions of the voltage. However, three tries should be enough.

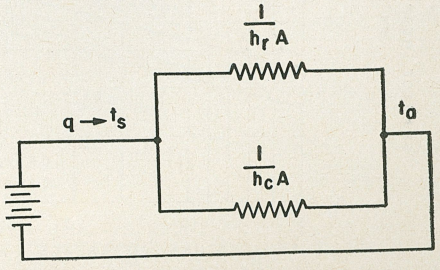


Figure 1. Thermal circuit for tube envelope

Paper 51-227, recommended by the AIEE Electronics Committee and approved by the AIEE Technical Program Committee for presentation at the AIEE Summer General Meeting, Schenectady, N. Y., June 25-29, 1951. Manuscript submitted March 28, 1951; made available for printing May 21, 1951.

MRS. B. O. BUCKLAND is with the General Electric Company, Schenectady, N. Y.

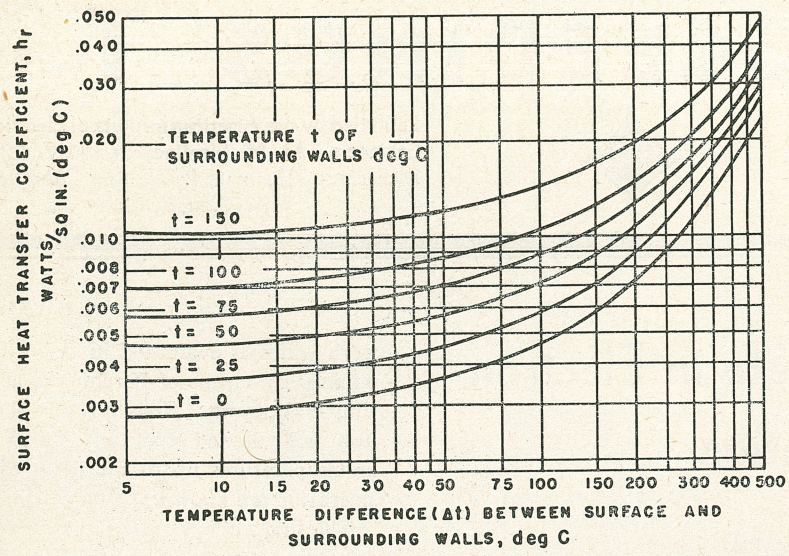


Figure 2. Surface heat-transfer coefficient for radiation, with emissivity 0.9

Radiation

The surface heat-transfer coefficient for radiation h_r is a way of expressing the Stefan-Boltzmann law for the radiation between two bodies at different temperatures. The radiation coefficient given here is a useful tool, rather than a physical constant like the h_c, which can be measured. Heat transferred by radiation is proportional to the fourth power of the temperature; the net radiation between two surfaces, one of which is relatively small and surrounded by the other at a different temperature is given by the expression, with units in watts, inches, and degrees centigrade

$$\frac{q_r}{A} = 37.1\, e_1 \left[\frac{(T_1)^4}{1{,}000} - \frac{(T_2)^4}{1{,}000}\right]$$

where

e_1 = emissivity of surface of small body, dimensionless

T_1, T_2 = absolute temperatures of the two surfaces, degrees Kelvin = 273 + degrees centigrade

In order to get this expression into the form most convenient for natural convection cooling, namely

$$q = (h_c + h_r) A\, \Delta t$$

the radiation coefficient h_r is defined as $q_r/A\,\Delta t$. This is the function plotted in Figure 2. For convenience in the use of this function, it is plotted for $e = 0.9$, a value which is a fair average for glass, for painted surfaces, and for well-oxidized metals. For cases where the surface emissivity is known accurately, the heat-transfer coefficient is adjusted by multiplying by the ratio of the known emissivity to 0.9.

Free Convection

The free convection coefficient for air is plotted in Figure 3. The basic equations, which are to be found in McAdams,[2] are plotted here in electrical units.

The free convection curves are drawn for air at atmospheric pressure with average temperature of surface and ambient 65 degrees centigrade. To correct for other pressures, as in high-altitude operation, the coefficients must be multiplied by the square root of the ratio of the operating pressure to standard atmosphere. To correct for other temperatures, a nearly linear factor is applied, 1.06 for arithmetic mean temperature of surface and ambient equal to zero degrees centigrade, and 0.95 for arithmetic mean of 150 degrees centigrade.

For other fluids, no simple general and precise relation will suffice, because of the wide variation of properties among fluids, and also the variation with temperature of the properties of any one fluid. However, useful correlations are available in the book by W. H. McAdams,[2] page 243 and page 248. An indication of the order of magnitude to be expected is shown by Figure 14, which gives values for water in ducts.

Fins

If calculations on the basis of Figures 2 and 3 indicate that the tube wall will run only a little too hot, some expedients may be tried before forced cooling is resorted to. One possibility is the use of fins, or other extended surfaces, where the tube configuration will allow their construction.

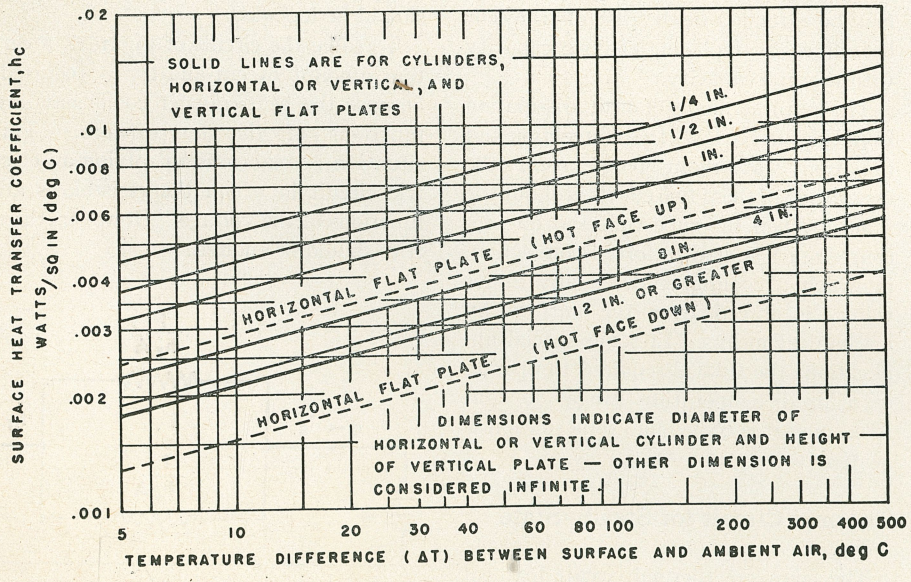


Figure 3. Surface heat-transfer coefficient for free convection in air, cylinders and flat plates

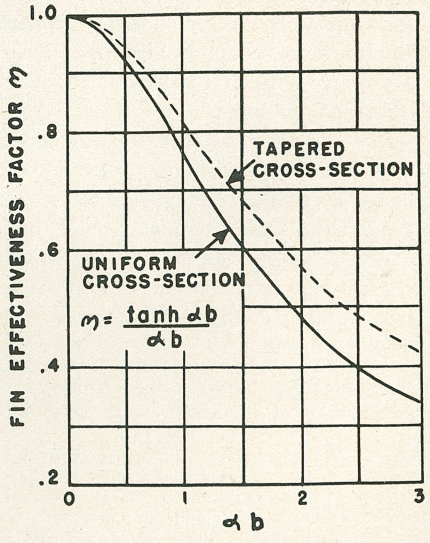


Figure 4. Effectiveness of fins on flat surfaces

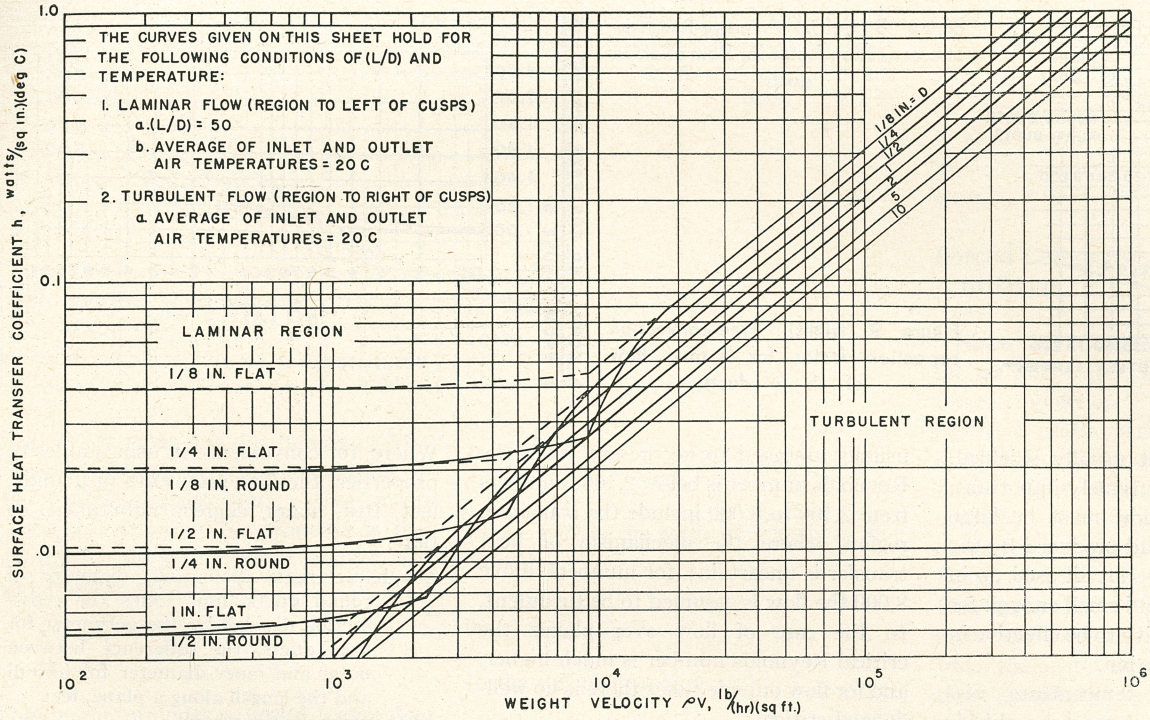


Figure 5. Forced-convection coefficient for air in smooth straight ducts

The additional heat dissipated by fins is given by the relation:

$$q = hA\eta(t_w - t_a) \tag{2}$$

where η, the fin effectiveness for fins of uniform section, is given by Harper and Brown[3] as:

$$\eta = \frac{\tanh \alpha b}{\alpha b} \tag{3}$$

where

$$\alpha = \sqrt{\frac{hA_s}{kA_0 b}} \tag{4}$$

which for flat-sided fins becomes:

$$\alpha = \sqrt{\frac{2h}{k\delta}} \tag{5}$$

In this case

h = surface heat transfer coefficient
k = thermal conductivity of fin material
δ = fin thickness
A_s = fin surface area
A_0 = fin cross-section area
b = fin height

Figure 4 gives the numerical value of effectiveness for fins of uniform cross-section. It also gives values for tapered fins, where the fin thickness in the above expression for α is the average of base and tip thickness. Effectiveness for a variety of other configurations is given by Gardner.[4]

Several factors must be considered in the use of fins. In the first place, the fin characteristic known as "fin effectiveness" must be high, because this is the quantity that determines how closely the fin surface temperature approaches the fin base temperature. Also, the radiation part h_r of the heat-transfer coefficient applies only to the envelope of the heat-dissipating area. The usual values of free convection coefficient h_c are valid only if the spacing between fins is great enough for free convection to take place effectively. A rule-of-thumb states that free convection tends to be seriously suppressed with spacings less than a quarter of an inch. Recent data[5] give the actual thickness of the boundary layer as a function of the length of a vertical fin; the example of the reference states that for a wall at 93 degrees centigrade in a 20 degree centigrade ambient, 8 inches from the base the thickness is 0.545 inch. In order for the two adjacent boundary layers not to interfere at all, the spacing for fins this height should exceed 1.09 inch. However, somewhat closer spacing is allowable as the temperatures and velocities are dying away rapidly before this value is reached.

Chimneys

Another possibility of increasing the natural convection lies in the use of a shield to introduce a chimney effect. Some tubes are shielded for electronic reasons; the efficacy of the shielding will not be impaired by coating the shield with material of high emissivity, and the heat dissipation will be increased. Moreover, the air space between tube and shield is normally small, so that convection does not take place; opening up the gap to permit air flow will result in further cooling. The same possibility is present in

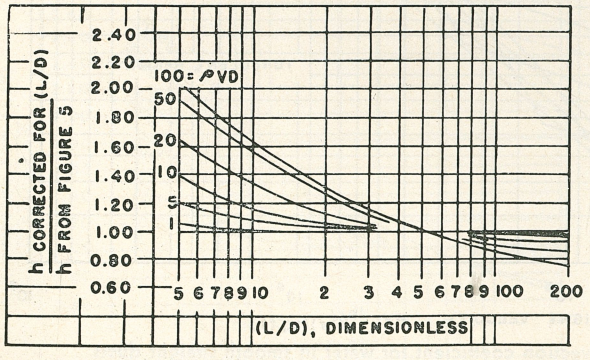


Figure 6 (left). L/D correction factor for laminar flow of air in round ducts

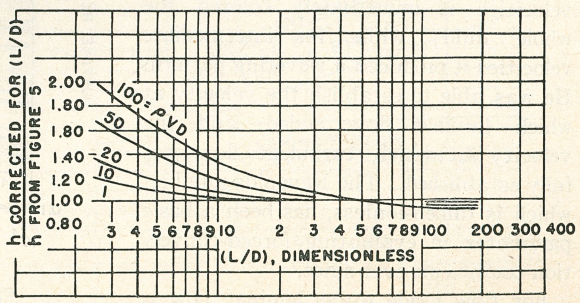


Figure 7 (right). L/D correction factor for laminar flow of air in flat ducts

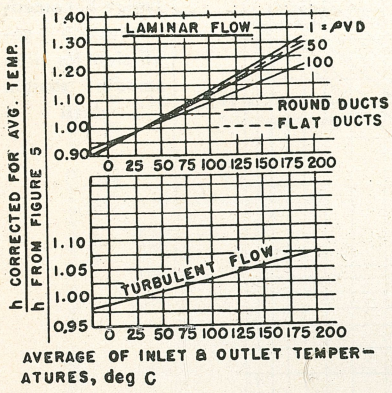


Figure 8 (left). Average-temperature correction factor for flow of air in ducts

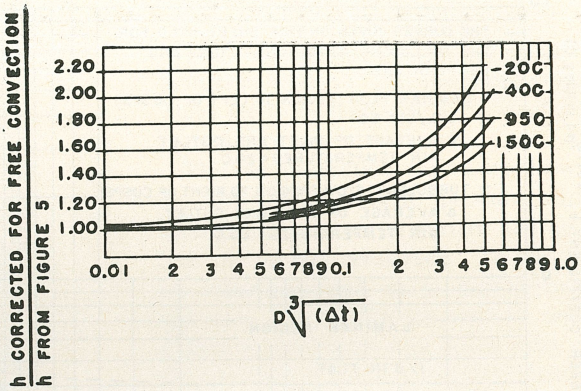


Figure 9 (right). Free-convection correction factor for laminar flow of air in ducts

the case of tubes not usually shielded. Here the spacing is extremely important, as the area for air flow must be large enough so that the head produced by the difference in density is not all used up in overcoming friction, and small enough for the increased air velocity to be effective in increasing heat dissipation.

To evaluate the temperature distribution in a tube enclosed by a shield, refer to Figure 1. An air resistance δ/kA is to be inserted ahead of t_s. This resistance is normally high, because free convection is absent, and the conductivity of still air is very low. Also, for an unpainted metal shield, the h_r is small. If the air space is opened up enough for δ/kA to be replaced by an additional $1/h_c$, and if the shield surface is painted so that h_r increases approximately 5-fold, the temperature rise of the tube will be lower, or for the same tube temperature the losses can be higher.

Forced Convection

The values of forced convection heat-transfer coefficients depend on the character of the flow, whether laminar, turbulent, or in the transition region between these two well-defined types. The basic work on character of flow was done by Osborne Reynolds,[6] and the Reynolds number which bears his name gives the criterion for flow. He studied the effect of varying water velocity in a straight glass tube, introducing ink at the center, and photographing the flow pattern. For low velocities the ink proceeded down the tube in a straight line; for high velocities it immediately colored the whole fluid; while for intermediate velocities it produced a wavering pattern. He was able to establish the velocity at which laminar flow ended, and the velocity at which turbulent flow was fully established. The Reynolds number, which is dimensionless, has been a basic parameter in evaluating forced-convection coefficients ever since.

For flow inside ducts laminar flow is usually assumed to be present when the Reynolds number is below 2,100; numbers from 2,100 to 8,000 include the transition region, where the mechanism of heat transfer is uncertain; for numbers above 8,000 the flow is assumed to be turbulent. In the case of flow over planes the critical Reynolds number is much higher, and for flow outside tubes there is no well-defined critical.

Three other dimensionless parameters, made up of thermal properties of the fluid, the velocity, and the characteristic dimension, have been found useful in expressing the heat-transfer coefficients. These parameters together with the Reynolds number are defined as follows:

Reynolds number: $\rho VD/\mu$
Stanton number: $h/c\rho V$
Prandtl number: $c\mu/k$
Heat-transfer factor: $\dfrac{h}{c\rho V}\left(\dfrac{c\mu}{k}\right)^{2/3}$

Where, for convenience in using tables of properties, the dimensions are in pounds, feet, Btu, hours, degrees Fahrenheit, so that

D = characteristic dimension, diameter of round ducts, four times the cross-section divided by the perimeter for flats ducts, the difference between outer and inner diameter for annuli and the length along a plane, feet
V = velocity, feet per hour
c = specific heat, Btu per pound-degree Fahrenheit, which is numerically equal to calories per gram-degree centigrade
k = thermal conductivity, Btu per hour-square foot per degree Fahrenheit per foot
ρ = density, pounds per cubic foot
μ = viscosity, pounds per hour-foot = 2.42 times centipoises
h = heat transfer coefficient, Btu per hour-square foot-degree Fahrenheit

General correlations of heat-transfer coefficients are available in the litera-

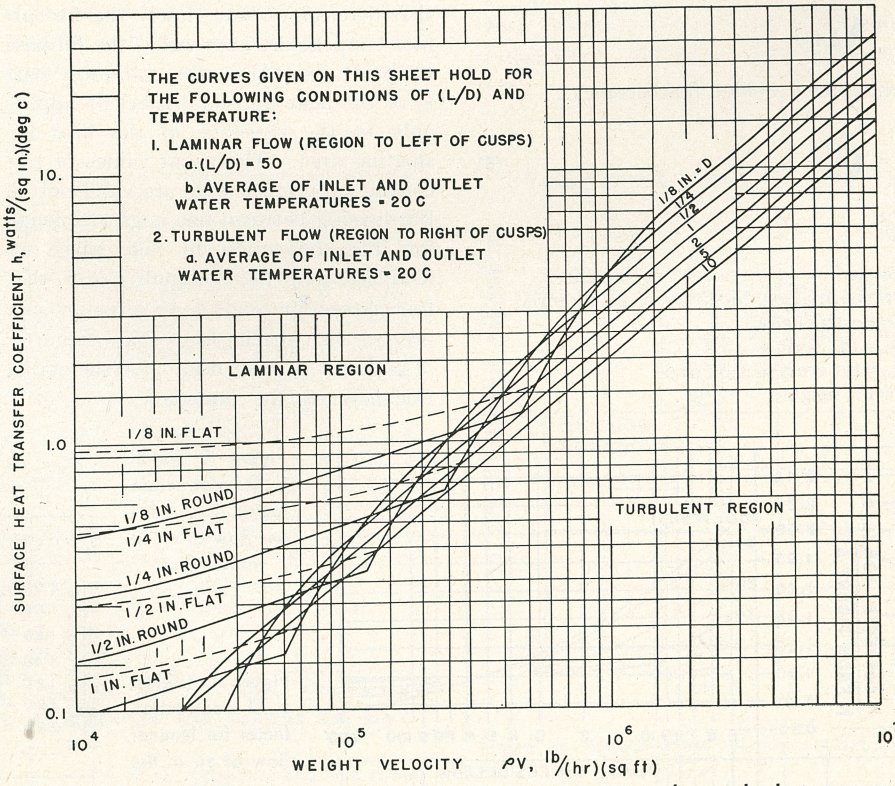

THE CURVES GIVEN ON THIS SHEET HOLD FOR THE FOLLOWING CONDITIONS OF (L/D) AND TEMPERATURE:
1. LAMINAR FLOW (REGION TO LEFT OF CUSPS)
a. (L/D) = 50
b. AVERAGE OF INLET AND OUTLET WATER TEMPERATURES = 20 C
2. TURBULENT FLOW (REGION TO RIGHT OF CUSPS)
a. AVERAGE OF INLET AND OUTLET WATER TEMPERATURES = 20 C
LAMINAR REGION
TURBULENT REGION
1/8 IN. FLAT
1/8 IN. ROUND
1/4 IN. FLAT
1/4 IN. ROUND
1/2 IN. FLAT
1/2 IN. ROUND
1 IN. FLAT
1/8 IN. = D
1/4
1/2
1
2
5
10
SURFACE HEAT TRANSFER COEFFICIENT h, watts/(sq in.)(deg c)
WEIGHT VELOCITY ρV, lb/(hr)(sq ft)

Figure 10. Forced-convection coefficient for water in smooth straight ducts

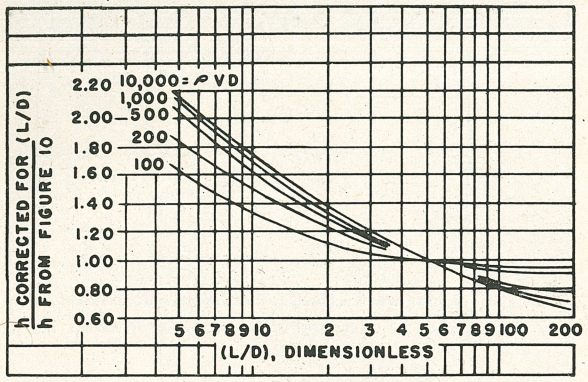


Figure 11 (left). L/D correction factor for laminar flow of water in round ducts

ture[7–10] for gases and for liquids of low viscosity. These correlations make use of the foregoing parameters. The most useful correlation so far proposed is the one due to Colburn,[11] which defines the heat-transfer factor j as:

$$j=\frac{h}{c\rho V}\left(\frac{c\mu}{k}\right)^{2/3}$$

This factor provides a useful means for correlating heat-transfer data on surfaces of different shapes; numerically, for many common shapes, it is approximately equal (within ±20 per cent) to the half-friction factor $f/2$, where f is the friction factor in the Fanning equation in hydraulics.

For gases in turbulent flow, there are three values of j in terms of Reynolds number R which have been determined by experiment to be as follows:

Flow in ducts, circular, flat, or annular:

$$j=\frac{0.021}{R^{0.2}}$$

Flow over planes: $j=\dfrac{0.0346}{R^{0.2}}$

Flow over banks of tubes: $j=\dfrac{0.33}{R^{0.4}}$

For liquids of low viscosity in ducts, in the laminar flow region ($R<2{,}100$) there are two values of j. Both are to be evaluated, and the higher one used:

$$j=\frac{1.615}{(L/D)^{1/3}R^{2/3}}$$

where L/D is the ratio of length to diameter

or $j=\dfrac{3.66}{(c\mu/k)^{1/3}R}$

In the turbulent region

$$j=\frac{0.025}{R^{0.2}}$$

For liquids of high viscosity, and for liquid metals, different relations for j exist. These relations are beyond the scope of this paper.

From the equation for j,

$$h=\frac{jc\rho V}{(c\mu/k)^{2/3}}$$

When h is expressed in watts per square inch-degrees centigrade, the dimensions of the right-hand number must come out in the same units, and this is most easily done by lumping all the conversion factors into a single constant, as follows:

$$h=0.00366\frac{jc\rho V}{(c\mu/k)^{2/3}}$$

where the units are tabulated in an earlier paragraph of this section.

It is to be expected that a general relationship applying to all fluids will be subject to error, and as a matter of fact heat-transfer coefficients determined from the general equations may be in error by ±30 per cent. However, being in error by ±30 per cent is better than being in error by 2-to-1 or 10-to-1.

Forced Convection for Air and Water in Ducts

Since fluid properties are well known for air and water, it is possible to plot forced-convection heat-transfer coefficients as watts per square inch-degrees centigrade, against weight velocity of the fluid. Figure 5 gives the heat-transfer coefficient for air in smooth straight ducts for the conditions specified on the figure, and is to be used with log-mean temperature difference. Figures 6, 7, and 8 give the correction factors to be applied when the operating conditions differ from those specified in the figure, while Figure 9 gives the free-convection correction factor.

Figure 10 gives the heat-transfer coefficient for water in smooth straight ducts. Figures 11, 12, and 13 give correction factors to be applied as in the case of air, except that the average of inlet and outlet water temperatures is 20 degrees centigrade. Figure 14 gives the curve for free-convection. In case of doubt as to which curve applies, determine h from both and use the higher.

Pressure Drop in Forced Convection

The relation between the half-friction factor $f/2$ and the heat transfer factor has already been discussed. Pressure drop in a pipe is given by the relation:

$$\Delta p=4f\frac{L}{D}\frac{\rho(V'')^2}{2g}$$

where

p = pressure drop, pound per square foot
f = Fanning friction factor, dimensionless

Figure 12 (below). L/D correction factor for laminar flow of water in flat ducts

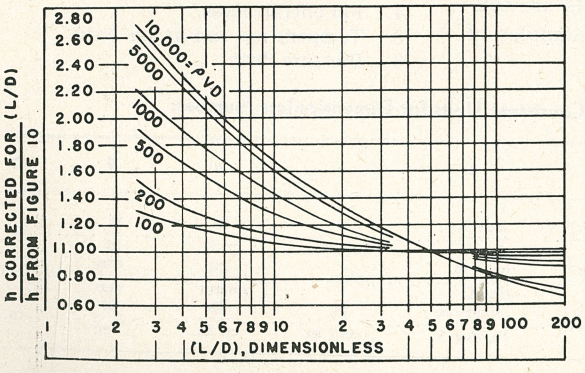
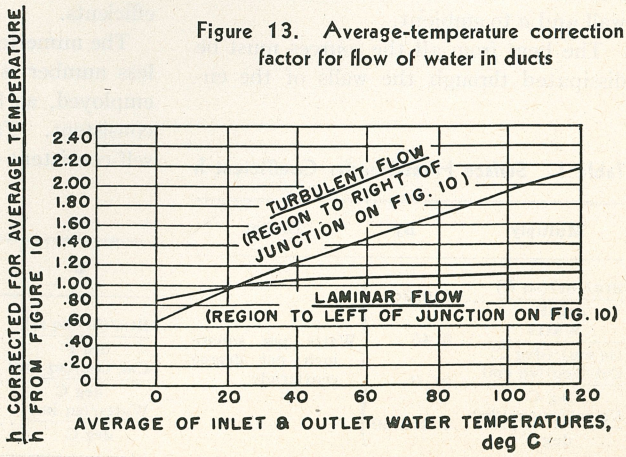


Figure 13. Average-temperature correction factor for flow of water in ducts



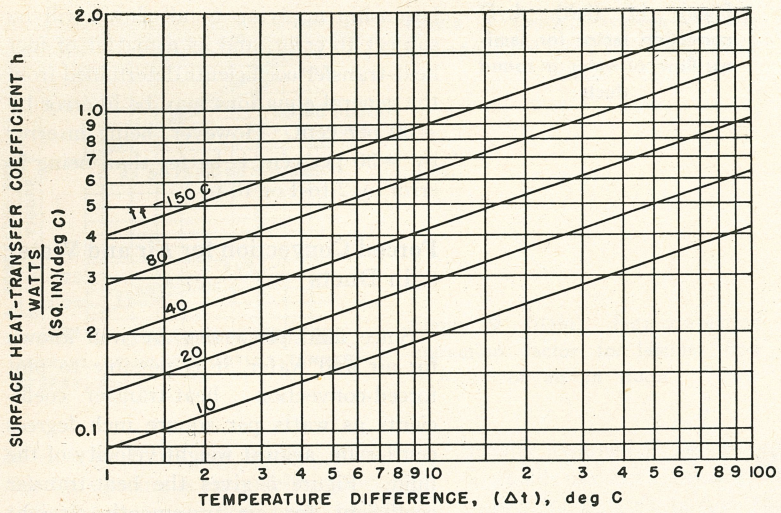


Figure 14 (left). Free-convection heat-transfer coefficient for water in ducts

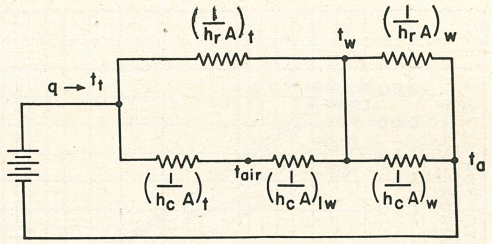


Figure 15 (above). Thermal circuit for tube in enclosure

L/D = length to diameter ratio, dimensionless
ρ = density, pound per cubic foot
V'' = velocity, feet per second
g = 32.2 feet per second per second

For turbulent flow the friction factor is very nearly constant, which means that for a given configuration the pressure drop varies as the square of the velocity. Since the heat-transfer coefficient varies as the 0.8 power of the velocity, it is obvious that high heat transfer can only be obtained at the expense of pressure drop. In deciding what velocities to use, the balance must be economic, that is, the size of blower or pump must be balanced against the gain in heat transfer.

Enclosure Ventilation

In many cases the tube with associated equipment is enclosed in a container. The question then arises as to how much hotter the tube will run. If the container is tightly closed, and dissipating heat by radiation and free convection, the temperature rise may be estimated from Figures 2 and 3. The thermal circuit is shown in Figure 15, where subscript t refers to tube, iw to inner wall, w to outer wall and a to ambient.

The heat from all the sources must be dissipated through the walls of the enclosure. The first step is to evaluate the wall temperature, which is done as before, by cut-and-try. The surface temperature is independent of the sources within the enclosure, except perhaps for local heating due to close proximity of one of the sources. When the surface temperature has been evaluated, the tube temperature and the box air temperature may be calculated. Radiation goes directly from the tube to the wall, while convection heats the air and must, therefore, be evaluated for both surfaces.

This calculation will give the maximum tube temperature. Opening up the box by louvers or holes at top and bottom edges will produce a chimney effect which will lessen the temperatures. The chimney effect is a function of the height of the opening above the heat source, and is, therefore, less effective if holes are punched throughout the walls.

Conversion Factors

Data on heat-transfer coefficients and on fluid properties are to be found in many sources, expressed in a variety of units. Table I gives the principal conversion factors used for heat transfer coefficients.

The numerical value of any dimensionless number is independent of the units employed, as long as the units are self-consistent. Table II gives examples of self-consistent units.

Table I. Surface Heat-Transfer Coefficient h

Multiply	By	To Get
Btu/(hr)(sq ft)/deg F	0.00366	Watts per square inch per degree centigrade
Watts/(sq cm)(deg C)	6.45	
Cal/(sec)(sq cm)/deg C	27.0	
Kg cal/(sq m)(hr)/deg C	0.000750	

Table II. Self-Consistent Units for Dimensionless Numbers

h	k	c	μ	ρ	V	D
Btu/(hr)(sq ft)/deg F	Btu/(hr)(sq ft)/deg F/ft	Btu/lb/deg F	lb/(hr)(ft)	lb/cu ft	ft/hr	ft
Cal/(sec)(sq cm)/deg C	cal/(sec)(sq cm)/deg C/cm	cal/gm/deg C	poise	gm/cu cm	cm/sec	cm
Watts/(sq in.)/deg C	watts/(sq in.)/deg C/in.	watt-sec/lb/deg C	lb/(sec) in	lb/cu in.	in./sec	in.

The most troublesome unit is viscosity. If its value is given in centipoises, multiply it by 2.42 to get pounds per hour-foot. If it is given in centistokes, it is a kinematic viscosity, and must be multiplied by the specific gravity to get centipoises. If it is given in pounds force-second per square foot, multiply it by 116,000 to get pounds per hour-foot.

Appendix I. The 892-R Triode as an Example

Figure 16 shows the construction of the *892-R* triode. Looked at as a heat-dissipating device rather than as an electron-producing device, the principal source of heat is the anode, with some filament loss, Although the plate dissipation rating is 4 kw-a test was run under the following conditions: plate dissipation 5 kw and filament 1.32 kw, making a total of 6,320 watts to be dissipated by the finned radiator. For this condition, the test air flow was 450 cubic feet per minute. The problem is to determine the temperature which would be read by a thermocouple embedded in the radiator, with inlet air at 27 degrees centigrade.

There are 48 copper fins, 9 inches long, average height 2.44 inches, thickness 0.072 inch, on a core 2.625 inches outside diameter. These dimensions determine the fin spacing to be 0.100 inch at the base and 0.418 inch at the tip.

The quantities to be evaluated are as follows:

1. Air temperature rise.
2. Equivalent diameter of cooling ducts.
3. Surface heat-transfer coefficient.
4. Fin effectiveness.
5. Temperature rise at fin base.
6. Pressure drop.

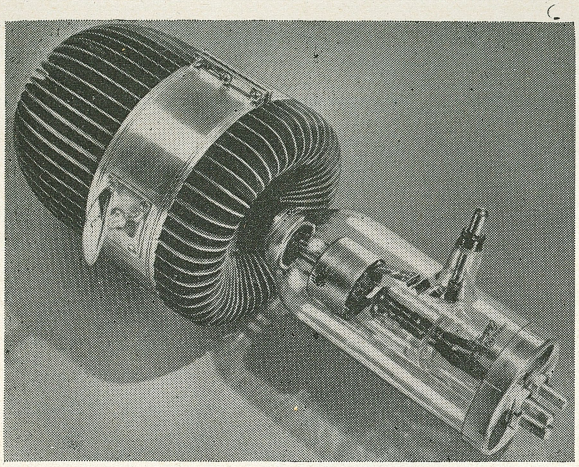


Figure 16. Triode used in heat transfer and pressure drop calculations

To find the air temperature rise Δt, the steady state input is divided by the product of weight and specific heat of the air. The weight flow in pounds per second is $^1/_{60}$ of the flow in cubic feet per minute times the density, where standard air density is 0.075 pounds per cubic foot. The specific heat of air is 0.24 calorie per gram per degree centigrade; to get it in watt seconds per pound per degree centigrade, multiply by 1,899, obtaining 455. The temperature rise is then given by

$$t=\frac{q}{cw}=\frac{6{,}320\times 60}{455\times 450\times 0.075}=24.7C$$

The area for flow is

$$2.44(0.100+0.418)/2=0.631 \text{ square inch}$$

Equivalent diameter is

$$4\times 0.631/(4.88+0.518)=0.467 \text{ inch}$$

Flow velocity V is

$$\frac{450\times 144\times 60}{48\times 0.631}=128{,}500 \text{ feet per hour}$$

Weight velocity ρV is

$$0.075\times 128{,}500=9{,}650 \text{ pounds per hour per square feet}$$

The heat-transfer coefficient is found from Figure 5, using equivalent diameter 0.467 inch and weight velocity 9,650, to be 0.0360 watt per square inch per degree centigrade.

Fin effectiveness is evaluated from equations 3 and 5, using k of copper as 9.75 watts per square inch-degrees centigrade per inch.

$$\alpha b=b\sqrt{\frac{2h}{k\delta}}=2.44\sqrt{\frac{2\times 0.0360}{9.75\times 0.072}}=0.783$$

From Figure 4, read fin effectiveness η as 0.84.

The temperature rise at the fin base now is determined from equation 2, with the finned area having an effectiveness of 0.84, and the unfinned area 1.0.

$$\Delta t=\frac{6{,}320}{0.0360[0.84(48\times 2\times 2.44\times 9.0)+1.0(2.625\pi-48\times 0.072)9.0]}=96.9 \text{ C}$$

The hot spot temperature is then $96.9+22.1+27=146$ degrees centigrade. The measured test value was 128 degrees centigrade rise above 27 degrees centigrade ambient, or 155 degrees. It may seem that the calculated value is thus not conservative. The low value may be explained on the basis that the heat loss is assumed to be uniformly distributed within the core, while in fact it is lower toward the ends and higher toward the center.[12]

A conservative value of pressure drop is obtained by using the friction factor of 0.0137 which is the maximum for turbulent flow in ducts. Substituting in the equation for pressure drop, and converting to inches of water by multiplying by 0.193, the pressure drop is 0.25 inch of water. This value does not include entrance and exit losses, which increase the required head to 0.5 inch of water.

References

1. Treatise on Electricity and Magnetism, **J. C. Maxwell.** Oxford University Press, London, England, 1892, Third Edition, volume I, pages 364–65.

2. Heat Transmission, **W. H. McAdams.** McGraw-Hill Book Company, New York, N. Y., 1942.

3. Mathematical Equations for Heat Conduction in the Fins of Air-Cooled Engines, **Harper, Brown.** *Report 158*, National Advisory Committee for Aeronautics.

4. Efficiency of Extended Surface, **K. A. Gardner.** *Transactions*, The American Society of Mechanical Engineers (New York, N. Y.), volume 67, 1945, Number 8, page 621.

5. Introduction to the Transfer of Heat and Mass, **E. Eckert.** McGraw-Hill Book Company, New York, N. Y., 1950, page 171.

6. Scientific Papers of Osborne Reynolds, **O. Reynolds,** London, England, 1901, page 81.

7. Laminar Heat-Transfer Coefficients for Ducts, **R. H. Norris, D. D. Streid.** *Transactions*, The American Society of Mechanical Engineers (New York, N. Y.), volume 60, 1940, page 525.

8. High Performance Fins for Heat Transfer, **R. H. Norris, W. A. Spofford.** *Transactions*, The American Society of Mechanical Engineers (New York, N. Y.), volume 62, 1942, page 489.

9. Basic Laws and Data of Heat Transmission—Forced Convection, **W. J. King.** *Transactions*, The American Society of Mechanical Engineers (New York, N. Y.), volume 50, 1932, page 410.

10. Surface Heat Transfer Coefficients for Hydrogen-Cooled Rotating Electric Machines, **D. S. Snell, R. H. Norris, Mrs. B. O. Buckland.** AIEE Transactions, volume 69, 1950, pages 174–85.

11. A Method of Correlating Forced Convection Data and a Comparison with Fluid Friction, **A. P. Colburn.** *Transactions*, American Institute of Chemical Engineers (New York, N. Y.), volume 29, 1933, page 174.

12. Temperature Distribution in Vacuum Tube Coolers with Forced Air Cooling, **I. E. Mouromtseff.** *Journal of Applied Physics* (New York, N. Y.), volume 12, 1941, page 491.

No Discussion